Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2022
Deferred Examinations

Marking Scheme

Applied Mathematics

Higher Level

Note to teachers and students on the marking schemes for the deferred examinations

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. However, it should be noted that the marking schemes for the deferred examinations may not necessarily be as detailed as the corresponding marking schemes for the main sitting of an examination, which serve to ensure consistency across a large team of examiners.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination, and the need to maintain consistency in standards between the main sitting and the deferred sitting and from year to year. In the case of the deferred examinations, this means that the level of detail may vary by question, as the marking scheme will only have been finalised for the questions attempted by the candidates who sat these examinations.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with a senior examiner when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes (whether for the main examinations or the deferred examinations) should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination concerned. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination will not necessarily be the same for the deferred sitting as for the main sitting or from one year to the next.

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: (Att 2).

2 The marking scheme shows one correct solution to each question.
In many cases there are other equally valid methods.

1. (a) (i)

$$S_A = 4t + \frac{1}{2}(2)t^2 \quad (5)$$

$$S_B = 5(t-3) + \frac{1}{2}(4)(t-3)^2 \quad (5)$$

$$S_A + S_B = 1143$$

$$4t + t^2 + 5(t-3) + 2(t-3)^2 = 1143$$

$$t = 20 \quad (5)$$

(ii)

$$S_A = 4(20) + \frac{1}{2}(2)(20)^2$$

$$S_{\boldsymbol{A}} = 480 \quad (5)$$

(iii)

$$S_A = 4t + \frac{1}{2}(2)t^2 = 480 - 160$$

$$t^2 + 4t - 320 = 0$$

$$t = 16 \quad (5)$$

$$S_B = 5(16-3) + \frac{1}{2}(4)(16-3)^2 = 403$$

$$d = 1143 - 320 - 403 = 420 \text{ m} \quad (5) \qquad (30)$$

(b) (i)

$$s = ut + \frac{1}{2}at^2$$

$$h = 0 + \frac{1}{2}(9.8)t^2 \quad (5)$$

$$h - \frac{15}{64}h = 0 + \frac{1}{2} \times (9.8) \times (t-1)^2 \quad (5)$$

$$\frac{49}{64} \times \frac{1}{2}(9.8)t^2 = 0 + \frac{1}{2} \times (9.8) \times (t-1)^2$$

$$t = 8 \text{ s} \quad (5)$$

(ii)

$$h = 0 + \frac{1}{2}(9.8)(8)^2$$

$$h = 313.6 \text{ m.} \quad (5) \qquad (20)$$

2. (a) (i)

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

$\vec{V}_{AB} = (15\cos\alpha \ \ \vec{\imath} + 15\sin\alpha \ \ \vec{\jmath}) - (-13\cos\beta \ \ \vec{\imath} + 13\sin\beta \ \ \vec{\jmath})$

$\vec{V}_{AB} = (9 + 5) \ \ \vec{\imath} + (12 - 12) \ \vec{\jmath}$

$\vec{V}_{AB} = 14 \ \ \vec{\imath} + 0 \ \vec{\jmath}$ (5)

$t = \frac{|AB|}{14} = \frac{42}{14} = 3$ (5)

(ii)

$\vec{r}_C = (42 + 25)\,\vec{\imath} + (22t)\,\vec{\jmath}$

$= 67 \ \vec{\imath} + 66 \ \vec{\jmath}$ (5)

$\vec{r}_A = 15\cos\alpha \times t \ \ \vec{\imath} + 15\sin\alpha \times t \ \ \vec{\jmath}$

$= 9t \ \ \vec{\imath} + 12t\,\vec{\jmath}$

$= 27 \ \vec{\imath} + 36 \ \vec{\jmath}$ (5)

$\vec{r}_{CA} = 40 \ \ \vec{\imath} + 30 \ \ \vec{\jmath}$

$d \ \ = \sqrt{40^2 + 30^2}$

$= 50 \text{ km}$ (5) (25)

(b) (i)

$3\sin\alpha \times t = 72$ (5)

$x = (5 - 3\cos\alpha)t$ (5)

$x = \frac{(5-3\cos\alpha)\times 72}{3\sin\alpha} = \frac{24(5-3\cos\alpha)}{\sin\alpha}$

$\frac{dx}{d\alpha} = \frac{\sin\alpha(72\sin\alpha) - 24(5-3\cos\alpha)\cos\alpha}{(\sin\alpha)^2} = 0$ (5)

$72\sin^2\alpha = 120\cos\alpha - 72\cos^2\alpha$

$\alpha = \cos^{-1}\left(\frac{3}{5}\right) = 53.13°$ (5)

(ii)

$\text{time} = \frac{72}{3\sin\alpha}$

$= 30 \text{ s}$ (5) (25)

3. (a)

$$14\sin\alpha \times t - \tfrac{1}{2}gt^2 = 0 \qquad (5)$$

$$t = \frac{28\sin\alpha}{g}$$

$$14\cos\alpha \times t = 10 \qquad (5)$$

$$14\cos\alpha \times \frac{28\sin\alpha}{g} = 10 \qquad (5)$$

$$\sin 2\alpha = \tfrac{1}{2}$$

$$\alpha = 15° \ \text{ or } \ 75° \qquad (5) \qquad (20)$$

(b) (i)

$$r_j = 0 \qquad (5)$$

$$u\sin\beta \times t - \tfrac{1}{2}g\cos 45 \ \times t^2 = 0$$

$$t = \frac{2u\sin\beta}{g\cos 45} = \frac{2\sqrt{2}u\sin\beta}{g} \qquad (5)$$

(ii)

$$R = u\cos\beta \times t - \tfrac{1}{2}g\sin 45 \ \times t^2 \qquad (5)$$

$$= u\cos\beta \times \frac{2\sqrt{2}u\sin\beta}{g} - \tfrac{1}{2}g\sin 45 \ \times \left(\frac{2\sqrt{2}u\sin\beta}{g}\right)^2$$

$$= \frac{2\sqrt{2}u^2}{g} \times \sin\beta\cos\beta - \frac{2\sqrt{2}u^2}{g} \times \sin^2\beta$$

$$= \frac{2\sqrt{2}u^2}{g} \times \sin\beta\,(\cos\beta - \sin\beta) \qquad (5)$$

(iii)

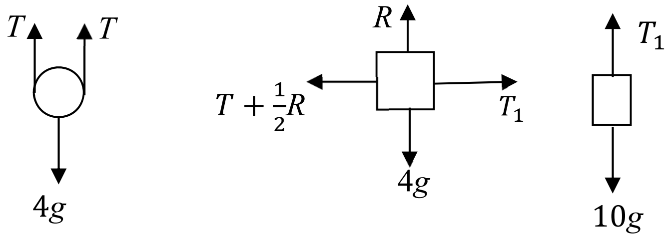
$$v_{\vec{i}} = u\cos\beta - g\sin 45 \times t$$
$$v_{\vec{i}} = u\cos\beta - 2u\sin\beta$$

$$v_{\vec{j}} = u\sin\beta - g\cos 45 \times t$$
$$v_{\vec{j}} = -u\sin\beta \qquad (5)$$

$$\tan 45 = \frac{-v_{\vec{j}}}{v_{\vec{i}}}$$
$$u\cos\beta - 2u\sin\beta = u\sin\beta$$
$$\tan\beta = \tfrac{1}{3}$$

$$\beta = \tan^{-1}\tfrac{1}{3} = 18.4° \qquad (5) \qquad (30)$$

4. (a)



$$2T - 4g = 4a \quad (5)$$

$$T_1 - T - 2g = 4 \times 2a \quad (5)$$

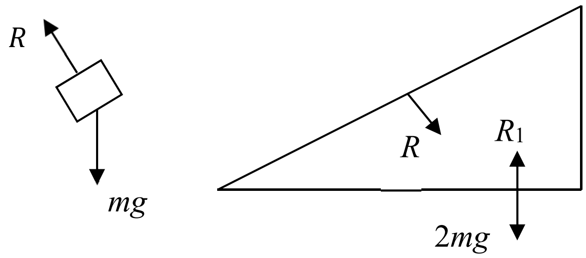
$$10g - T_1 = 10 \times 2a \quad (5)$$

$$a = \frac{1}{5}g \quad (5)$$

$$a_P = \frac{1}{5}g$$

$$a_Q = a_R = \frac{2}{5}g \quad (5) \qquad (25)$$

(b) (i)



(5)

(ii) $2m$ $$R \sin 45 = 2mq \quad (5)$$

$$R = 2\sqrt{2}mq$$

m $$mg\cos 45 - R = mq\sin 45 \quad (5)$$

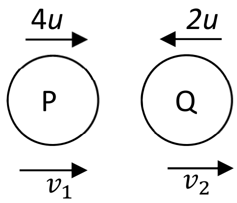
$$mg\cos 45 - 2\sqrt{2}mq = mq\sin 45$$

$$q = \frac{1}{5}g \quad (5)$$

(iii) m $$mg\sin 45 = m(p - q\cos 45)$$

$$p = \frac{6}{5\sqrt{2}}g \quad (5) \qquad (25)$$

5. (a)



(i) PCM $3m(4u) + 5m(-2u) = 3mv_1 + 5mv_2$ (5)

NEL $v_1 - v_2 = -e\big(4u - (-2u)\big)$ (5)

$$3v_1 + 5v_2 = 2u$$
$$v_1 - v_2 = -6eu$$

$$v_1 = \frac{u(1-15e)}{4} \qquad v_2 = \frac{u(1+9e)}{4}$$ (5), (5)

(ii)

$$v_2 > 0 \quad \Rightarrow \quad v_1 > 0$$

$$1 - 15e > 0$$
$$0 \le e < \frac{1}{15}$$ (5) (25)

(b) (i)

A	m	$u\cos\alpha\ \vec{\imath} + u\sin\alpha\ \vec{\jmath}$	$v_1\ \vec{\imath} + u\sin\alpha\ \vec{\jmath}$
B	m	$0\ \vec{\imath} + 0\ \vec{\jmath}$	$v_2\ \vec{\imath} + 0\ \vec{\jmath}$

PCM $mu\cos\alpha + m(0) = mv_1 + mv_2$ (5)

NEL $v_1 - v_2 = -eu\cos\alpha$ (5)

$$v_1 + v_2 = u\cos\alpha$$
$$v_1 - v_2 = -eu\cos\alpha$$
$$v_1 = \left(\frac{1-e}{2}\right)u\cos\alpha$$ (5)
$$v_2 = \left(\frac{1+e}{2}\right)u\cos\alpha$$

$$v_{\mathrm{A}} = \sqrt{\left\{\left(\frac{1-e}{2}\right)u\cos\alpha\right\}^2 + \{u\sin\alpha\}^2}$$
$$v_{\mathrm{B}} = \left(\frac{1+e}{2}\right)u\cos\alpha$$ (5)

(ii)

$$\tan 2\alpha = \frac{u\sin\alpha}{v_1} = \left(\frac{2}{1-e}\right)\tan\alpha$$
$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\tan\alpha}{1-e}$$
$$\tan^2\alpha = e$$ (5) (25)

6. (a) (i)

$$v = \omega\sqrt{A^2 - x^2}$$

$$8 = \omega\sqrt{A^2 - 3^2} \qquad (5)$$

$$6 = \omega\sqrt{A^2 - 4^2} \qquad (5)$$

$$\frac{8}{6} = \frac{\omega\sqrt{A^2-3^2}}{\omega\sqrt{A^2-4^2}}$$

$$A = 5 \text{ m} \qquad (5)$$

(ii)

$$8 = \omega\sqrt{5^2 - 3^2}$$

$$\omega = 2$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \text{ s} \qquad (5)$$

(iii)

$$x = A\cos\omega t$$

$$\frac{1}{2}A = A\cos 2t$$

$$t = \frac{\pi}{6} \text{ s.} \qquad (5) \qquad (25)$$

(b)

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh \qquad (5)$$

$$4gd = v^2 + 2g(d + d\cos\alpha)$$

$$v^2 = 2gd - 2gd\cos\alpha \qquad (5)$$

$$T + mg\cos\alpha = \frac{mv^2}{d} \qquad (5)$$

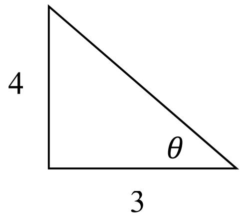
$$0 + mg\cos\alpha = \frac{mv^2}{d} \qquad (5)$$

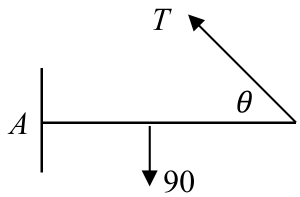
$$gd\cos\alpha = 2gd - 2gd\cos\alpha$$

$$3\cos\alpha = 2$$

$$\alpha = \cos^{-1}\left(\frac{2}{3}\right) \qquad (5) \qquad (25)$$

7. (a)





(i) $\circlearrowright A$ $\quad T\sin\theta \times 6 = 90 \times 3$ (5)

$\sin\theta = \frac{4}{5}$ (5)

$T \times \frac{4}{5} \times 6 = 270$

$T = 56.25$ N (5)

(ii) $Y + T\sin\theta = 90$

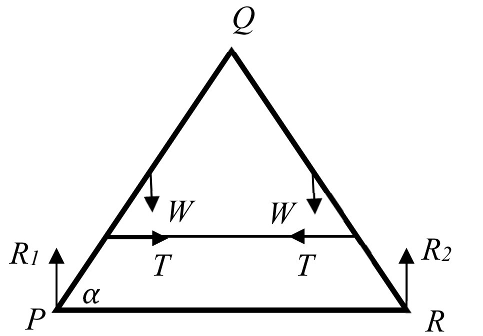
$Y = 90 - 45 = 45$ (5)

$X = T\cos\theta = 33.75$

$R = \sqrt{X^2 + Y^2}$

$R = \sqrt{33.75^2 + 45^2} = 56.25$ N (5) (25)

(b)



PQR $\circlearrowright P$

$R_2 \times 2l\cos\alpha = W \times \frac{1}{2}l\cos\alpha + W \times \frac{3}{2}l\cos\alpha$ (5), (5)

$R_2 = W$ (5)

$R_1 + R_2 = 2W$

$R_1 = W$

QR $\circlearrowright Q$

$R_2 \times l\cos\alpha = T \times \frac{3}{4}l\sin\alpha + W \times \frac{1}{2}l\cos\alpha$ (5)

$T \times \frac{3}{4}l\sin\alpha = W \times \frac{1}{2}l\cos\alpha$

$T = \frac{2W}{3\tan\alpha}$ (5) (25)

8. (a) Let M = mass per unit area

mass of element = $M\{2adx\}$

moment of inertia of the element = $M\{2adx\}\,x^2$ (5)

moment of inertia of the lamina = $2aM\int_{-a}^{a} x^2\,dx$ (5)

$= 2aM\left[\frac{x^3}{3}\right]_{-a}^{a}$ (5)

$= \frac{4}{3}Ma^4$

$= \frac{1}{3}ma^2$ (5) (20)

(b) (i) $I = \frac{1}{3}ma^2 + \frac{1}{3}ma^2 + m\left(\sqrt{2}a\right)^2 + m\left(2\sqrt{2}a\right)^2$ (5), (5)

$I = \frac{32}{3}ma^2$

$Mgh = mg\left(\sqrt{2}a\right) + mg\left(2\sqrt{2}a\right) = 3\sqrt{2}mga$ (5)

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

$$T = 2\pi\sqrt{\frac{\frac{32}{3}ma^2}{3\sqrt{2}mga}} = \frac{8\pi}{3}\sqrt{\frac{\sqrt{2}a}{g}}$$ (5)

(ii) $2\pi\sqrt{\frac{L}{g}} = \frac{8\pi}{3}\sqrt{\frac{\sqrt{2}a}{g}}$ (5)

$$2\pi\sqrt{\frac{8\sqrt{2}}{9g}} = \frac{8\pi}{3}\sqrt{\frac{\sqrt{2}a}{g}}$$

$$\frac{8\sqrt{2}}{9} = \frac{16\sqrt{2}a}{9}$$

$a = \frac{1}{2}$ (5) (30)

9. (a)

$$B = \rho V g = 1000 \times \frac{74}{100}\left\{\frac{4}{3}\pi \times 1^3\right\} g \quad (5)$$

$$W = \rho V g = 1280\left\{\frac{4}{3}\pi \times 1^3 - \frac{4}{3}\pi \times r^3\right\} g \quad (5)$$

$$W = B \quad (5)$$

$$1280\left\{\frac{4}{3}\pi \times 1^3 - \frac{4}{3}\pi \times r^3\right\} g = 1000 \times \frac{74}{100}\left\{\frac{4}{3}\pi \times 1^3\right\} g \quad (5)$$

$$\{1 - r^3\} = \frac{74}{128}$$

$$r = 0.75 \text{ m} \quad (5) \qquad (25)$$

(b) (i)

$$B_R = \frac{W \times 1}{10} = \frac{1}{10} W$$

$$B_S = \frac{W \times 1}{10} = \frac{1}{10} W \quad (5)$$

$\circlearrowright C$ $$T \times 7r \sin\alpha + B_R \times 2.5r \sin\alpha = 2W \times 4r \sin\alpha \quad (5)$$

$$T \times 7 + \frac{1}{10} W \times 2.5 = 2W \times 4$$

$$T = \frac{31}{28} \text{ W} \quad (5)$$

(ii)

$$R + B_S + B_R + T = 3W \quad (5)$$

$$R + \frac{1}{10} W + \frac{1}{10} W + \frac{31}{28} \text{ W} = 3W$$

$$R = \frac{237}{140} W = 1.7W \quad (5) \qquad (25)$$

10. (a) (i)

$\int dr = \int \frac{1}{1+t^2} dt$ (5)

$[r]_0^r = [\tan^{-1} t]_{\pi/4}^t$ (5)

$r = \tan^{-1} t - \tan^{-1} \frac{\pi}{4}$

$r = \tan^{-1} t - 0.67$ (5)

(ii)

$\int \frac{dy}{y+4} = \int \cos^2 3x \, dx$ (5)

$\int \frac{dy}{y+4} = \int \frac{1}{2}(1 + \cos 6x) dx$

$[ln(y+4)]_{-3}^y = \frac{1}{2}\left[x + \frac{1}{6}\sin 6x\right]_0^{\pi/6}$

$ln(y+4) - ln(1) = \frac{\pi}{12} - 0$

$y = -2.7$ (5) (25)

(b) (i)

$\int \frac{1}{v} dv = -\frac{1}{100}\int dt$ (5)

$[\ln v]_{80}^v = -\frac{1}{100}[t]_0^t$

$v = 80e^{-\frac{1}{100}t}$ (5)

(ii)

$\frac{ds}{dt} = 80e^{-\frac{1}{100}t}$ (5)

$\int ds = 80 \int e^{-\frac{1}{100}t} dt$

$[s]_0^s = 80 \times (-100)\left[e^{-\frac{1}{100}t}\right]_0^t$

$s = 8000\left(1 - e^{-\frac{1}{100}t}\right)$ (5)

(iii)

$v\frac{dv}{ds} = -\frac{1}{100}v$

$100\int dv = -\int ds$

$100[v]_{80}^v = -[s]_0^s$

$100(v - 80) = -s$

$v = 80 - \frac{1}{100}s$ (5) (25)